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Globalization for geometric partial comodules

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From partial actions to partial comodules

1995 Exel – Partial dynamical systems (i.e. partial actions of groups on C^*-algebras)

1998 Exel – Partial actions of groups on sets

1999 Abadie – Globalization for partial actions of (topological) groups on sets/topological spaces/C^*-algebras

2004 Kellendonk-Lawson – Globalization for partial actions of (topological) groups on sets/topological spaces

2004 Megrelishvili-Schröder – Partial actions of (topological) monoids and their globalization

2008 Caenepeel-Janssen – Partial (co)module algebras (i.e. partial (co)actions of Hopf algebras on algebras)

2010 Alves-Batista – Enveloping (co)actions of partial (co)module algebras

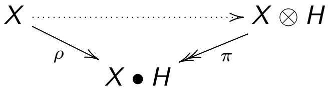
...

Geometric partial comodules

Let $(\mathcal{C}, \otimes, \mathbb{I})$ be a monoidal cat with pushouts and (H, Δ, ε) a coalgebra.

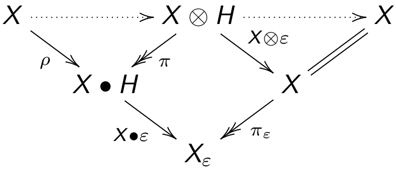
Definition [Hu-Vercruysse, 2018]

A geometric partial comodule (gpc_H) is a cospan

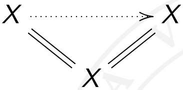


such that

- the following cospans coincide (up to isomorphism):



and



- ...

Geometric partial comodules

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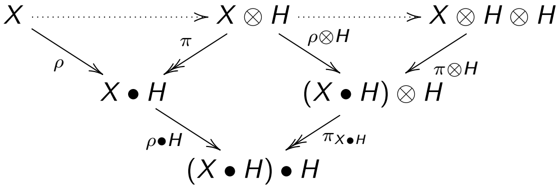
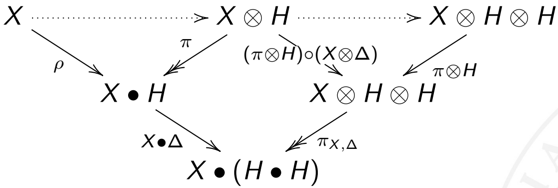
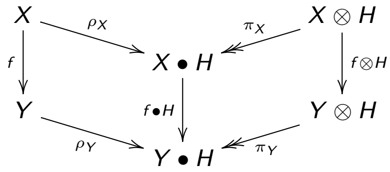




The category of geometric partial comodules

Definition

If $(X, X \bullet H, \pi_X, \rho_X)$ and $(Y, Y \bullet H, \pi_Y, \rho_Y)$ are gpcs$_H$, then a morphism of geometric partial comodules is a pair $(f, f \bullet H)$ of morphisms in $\mathcal{C}$ such that



commutes. We denote by gPCom^H the category of geometric partial comodules over H and their morphisms.

Examples of geometric partial comodules

Any (global) comodule with the structure $\mathcal{I}(Y, \delta) = (Y, Y \otimes H, id, \delta)$

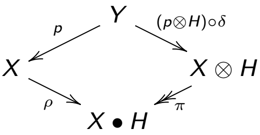
- Partial actions of groups/monoids on sets ($\mathcal{C} = \mathsf{Set}^{\mathsf{op}}$)
- Partial actions of (topological) groups/monoids on topological spaces ($\mathcal{C} = \mathsf{Top}^{\mathsf{op}}$)
- Partial modules over Hopf algebras ($\mathcal{C} = \mathsf{Vect}_{\Bbbk}^{\mathsf{op}}$)
- Partial comodules over Hopf algebras ($\mathcal{C} = \mathsf{Vect}_{\Bbbk}$)
- Partial comodule algebras over Hopf algebras ($\mathcal{C} = \mathsf{Alg}_{\Bbbk}$)
- Partial Hopf comodules over a bialgebra B ($\mathcal{C} = \mathsf{Mod}_B$)
- ...

Globalization for geometric partial comodules

Definition

Given a gpc_H $(X, X \bullet H, \pi, \rho)$, a globalization for X is an H-comodule (Y, δ) with a morphism $p : Y \to X$ in $\mathcal{C}$ such that

- the following diagram is a pushout square in $\mathcal{C}$



- Y is universal in the sense that the following is bijective

$$\mathrm{Com}^H(Z, Y) \to \mathrm{gPCom}^H(\mathcal{I}(Z), X), \qquad f \mapsto p \circ f\,.$$

X is globalizable if a globalization for X exists and we denote by gPCom^H_{gl} the full subcategory of the globalizable partial comodules.

When are geometric partial comodules globalizable?

Theorem [S.-Vercruysse]

Let H be a coalgebra in the monoidal category $\mathcal{C}$. Then a geometric partial comodule $(X, X \bullet H, \pi, \rho)$ is globalizable if and only if

- the following equalizer exists in Com^H

$$(Y_X, \delta) \xrightarrow{\kappa} (X \otimes H, X \otimes \Delta) \underset{(\pi\otimes H)\circ(X\otimes\Delta)}{\overset{\rho\otimes H}{\rightrightarrows}} (X \bullet H \otimes H, X \bullet H \otimes \Delta)$$

- the following is a pushout diagram in $\mathcal{C}$

$$\begin{array}{ccccc} & & Y_X & & \\ & \swarrow^{(X\otimes\varepsilon)\circ\kappa} & & \searrow^{\kappa} & \\ X & & & & X \otimes H \\ & \searrow_{\rho} & & \swarrow_{\pi} & \\ & & X \bullet H & & \end{array}$$

Under these equivalent conditions Y_X is the globalization of X.

Corollary

The functor $\mathcal{I} : \mathsf{Com}^H \to \mathsf{gPCom}^H$ corestricts to a fully faithful functor

$$\mathcal{J} : \mathsf{Com}^H \to \mathsf{gPCom}^H_{gl}.$$

Moreover, the assignment $X \mapsto Y_X$ induces a functor

$$\mathcal{G} : \mathsf{gPCom}^H_{gl} \to \mathsf{Com}^H$$

which is right adjoint to the fully faithful functor $\mathcal{J} : \mathsf{Com}^H \to \mathsf{gPCom}^H_{gl}$.

- Partial actions of groups and monoids on sets are globalizable [Abadie, Kellendonk-Lawson]
- Topological partial actions of topological groups on topological spaces are globalizable [Abadie]
- Partial comodule algebras over Hopf algebras are globalizable, but their globalization is not their enveloping coaction [Alves-Batista]
- In Top^{op}, $\mathsf{Alg}_{\Bbbk}$ and $\mathsf{CAlg}_{\Bbbk}$ a general globalization does not exist, that is $\mathsf{gPCom}^H_{gl} \subset \mathsf{gPCom}^H$.

Geometric partial comodules in abelian categories

Let $\mathcal{C}$ be any abelian monoidal category.

Theorem [S.-Vercruysse]

If H is a coalgebra in $\mathcal{C}$ s.t. Com^H admits equalizers and $\mathrm{Com}^H \to \mathcal{C}$ preserves them, then $\mathrm{gPCom}^H_{gl} = \mathrm{gPCom}^H$.

- Partial modules over Hopf algebras are globalizable and the globalization coincides with their standard dilation [Alves-Batista-Vercruysse]
- Partial representations of finite groups are globalizable [D'Adderio-Hautekiet-S.-Vercruysse]
- Partial comodules over Hopf algebras are globalizable
- Partial Hopf comodules over bialgebras are globalizable

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Many thanks

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